Aufgabe 1 a)

$$
\begin{aligned}
F(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(-(t/\tau)^2\right) \cdot \exp\left(i\omega t\right) dt \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(-(t/\tau)^2 + i\omega t\right) dt \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{\tau} - \frac{i\omega\tau}{2}\right)^2 - \left(\frac{\omega\tau}{2}\right)^2\right) dt \\
&= \frac{1}{\sqrt{2\pi}} \exp\left(-\left(\frac{\omega\tau}{2}\right)^2\right) \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{\tau} - \frac{i\omega\tau}{2}\right)^2\right) dt \\
&\underset{\text{Vorlesung}}{=} \frac{1}{\sqrt{2\pi}} \exp\left(-\left(\frac{\omega\tau}{2}\right)^2\right) \sqrt{\pi}\,\tau \\
&= \frac{\tau}{\sqrt{2}} \exp\left(-\left(\frac{\omega\tau}{2}\right)^2\right)
\end{aligned}
$$

Die Halbwertsbreite t_0 mit $f(t_0) = \frac{1}{2}$ ergibt sich zu $t_0 = 2\tau\sqrt{\ln 2}$
Die Halbwertsbreite ω_0 mit $F(\omega_0) = \frac{1}{2}F(0)$ ergibt sich zu $\omega_0 = \frac{4}{\tau}\sqrt{\ln 2}$

b)

$$
\begin{aligned}
A' = \mathcal{F}(A) &= \mathcal{F}(G_1 \otimes G_2) \\
&\underset{\text{Faltungssatz}}{=} \mathcal{F}(G_1) \cdot \mathcal{F}(G_2) \cdot \sqrt{2\pi} \\
&= \frac{\tau_1}{\sqrt{2}} \exp\left(-\left(\frac{\omega\tau_1}{2}\right)^2\right) \cdot \frac{\tau_2}{\sqrt{2}} \exp\left(-\left(\frac{\omega\tau_2}{2}\right)^2\right) \cdot \sqrt{2\pi} \\
&= \frac{\sqrt{\pi}\tau_1\tau_2}{\sqrt{2}} \cdot \exp\left(\frac{-\left(\omega\tau_1\right)^2 - \left(\omega\tau_2\right)^2}{4}\right) \\
&= \frac{\sqrt{\pi}\tau_1\tau_2}{\sqrt{2}} \cdot \exp\left(\frac{-\omega^2\left(\tau_1^2 + \tau_2^2\right)}{4}\right) \qquad \text{setze } \tau_3 = \sqrt{\tau_1^2 + \tau_2^2} \\
&= \frac{\sqrt{\pi}\tau_1\tau_2}{\sqrt{2}} \cdot \exp\left(-\left(\frac{\omega\tau_3}{2}\right)^2\right)
\end{aligned}
$$

$$\Rightarrow t_0 = 2\tau_3\sqrt{\ln 2} = 2\sqrt{\ln 2(\tau_1^2 + \tau_2^2)}$$

Aufgabe 2

$$
\begin{aligned}
\Delta\psi(\vec{r}, t) &= \left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \underbrace{\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}}_{=0,\text{ da } \psi \text{ unabhängig von } \theta \text{ und } \phi}\right) \frac{a}{r}\exp\left(ikr - i\omega t\right) \\
&= \left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\right)\frac{a}{r}\exp\left(ikr - i\omega t\right) \\
&= \left(\frac{1}{r^2}\frac{\partial}{\partial r}\right)\left(ikar\exp\left(ikr - i\omega t\right) - a\exp\left(ikr - i\omega t\right)\right) \\
&= \frac{1}{r^2}\left(-k^2 ar\exp\left(ikr - i\omega t\right)\right) \\
&= -\frac{k^2 a}{r}\exp\left(ikr - i\omega t\right) \\
\overset{!}{=} \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} &= -\frac{1}{v^2}\frac{a\omega^2}{r}\exp\left(ikr - i\omega t\right) \\
&= -\frac{k^2 a}{r}\exp\left(ikr - i\omega t\right)
\end{aligned}
$$